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UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET		
COURSE CODE	:	BGE 2113 311
COURSE	:	NUMERICAL METHOD
SEMESTER/SESSION	:	1-2024/2025
DURATION	:	3 HOURS

Instructions:

1. This booklet contains **9** questions. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 7 PRINTED PAGES INCLUDING COVER PAGE

INSTRUCTION: ANSWER ALL QUESTIONS. (100 MARKS)

QUESTION 1

Find the root of $f(x) = \sin x + \cos(1 + x^2) - 1$ over the interval $[1.5, 2.5]$ by using Secant method. Iterate until $|f(x_i)| \leq \varepsilon = 0.001$.

(7 marks)

QUESTION 2

Solve the following system of linear equations by using the Gauss elimination method.

$$
\begin{aligned}
2x_1 - 6x_2 - x_3 &= -38 \\
-3x_1 - x_2 + 7x_3 &= -34 \\
-8x_1 + x_2 - 2x_3 &= -20
\end{aligned}
$$

(9 marks)

QUESTION 3

Given that $f(x) = 3x + \sin x - e^x$ with $x_0 = 1$. Find the root of $f(x)$ by using Newton Raphson method. Iterate until $|f(x)| \leq 0.001$.

(7 marks)

QUESTION 4

Evaluate $\int_0^{0.6} 2e^{-1.5x}\,dx$ by using:

a) the Trapezoidal rule with step size, $h = 0.2$. (5 marks)

b) the $\frac{1}{3}$ Simpson's rule with subintervals, $n = 8$. (9 marks)

QUESTION 5

a) Approximate the values of the first derivative of the function $f(x) = 25x^3 - 6x^2 + 7x - 88$ at the point $x = 2.5$ with step size, $h = 0.25$ by using forward, backward and central differences. Do all computations in 4 decimal places.

(14 marks)

b) Given the actual value for $f'(2.5) = 445.75$. Find the relative error obtained in (a).

(4 marks)

QUESTION 6

Compute the linear least square polynomial for the following data:

x	0.8	2	3	4	6	8	8.5
y	1.2	1.95	2	2.4	2.5	2.7	2.6

(7 marks)

QUESTION 7

Solve the following system of linear equations by using LU factorization method.

$$
\begin{aligned}
2x_1 + 5x_2 + x_3 &= -5 \\
5x_1 + 2x_2 + x_3 &= 12 \\
x_1 + 2x_2 + x_3 &= 3
\end{aligned}
$$

if given $Y = \left(-5, \frac{49}{2}, \frac{13}{3}\right)^T$.

(14 marks)

QUESTION 8

Given that the differential equations $y' = -0.5y + 4e^{0.8t}$, $0 \le t \le 1$, $y(0) = 0$.

a) Use second order Runge-Kutta method with step size $h = 0.25$ to approximate the solution of the above initial value problem.

(14 marks)

b) Given the analytical solution is $y = \frac{4}{1.3}\left(e^{0.8t} - e^{-0.5t}\right) + 2e^{-0.5t}$. Find the absolute error obtained in (a).

(3 marks)

QUESTION 9

For a function $g(x)$, the Newton's divided differences are given as below.

i	x_i	$g(x_i)$	$g[x_{i-1},x_i]$	$g[x_{i-2},x_{i-1},x_i]$	$g[x_0,x_1,x_2,x_3]$
0	1.0	$g(x_0)$			
			$g[x_0,x_1]$		
1	2.7	$g(x_1)$		$g[x_0,x_1,x_2]$	
			$\frac{205}{21}$		0.0865
2	4.8	38.3		$\frac{2935}{1218}$	
			16.75		
3	5.6	51.7			

a) Determine the missing entries in the table above. (4 marks)

b) Estimate $g(3.5)$ with a third order Newton's divided difference. (3 marks)

......................................**END OF QUESTIONS**......................................

FORMULA

Absolute error: $= \left| Exact - Approximate \right|$

Relative Error: $= \left| \dfrac{Exact - Approximate}{Exact} \right|$

Newton Raphson method:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Secant method:

$$x_{i+1} = x_i - f(x_i)\left[\frac{x_{i-1} - x_i}{f(x_{i-1}) - f(x_i)}\right]$$

LU Factorization:

$$A = LU$$
$$LY = B$$
$$UX = Y$$

Newton's Divided Difference:

$$f_n(x) = f(x_0) + (x - x_0) f[x_0, x_1] + \cdots + (x - x_0)(x - x_1)\cdots(x - x_{n-1}) f[x_0, \cdots, x_{n-1}, x_n]$$

Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f_0 + f_n + 2\sum_{i=2}^{n-1} f_i\right]$$

Simpson's Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{i=1}^{n-1} f_i + 2\sum_{i=2}^{n-2} f_i\right]$$

$$\int_a^b f(x)\,dx \approx \frac{3h}{8}\left[f_0 + f_n + 3\sum_{i=1}^{n-1} f_i + 2\sum_{\substack{i=3 \\ \text{multiple of three}}}^{n-3} f_i\right]$$

Second order Runge-Kutta's method:

$$y_{i+1} = y_i + \frac{h}{2}(k_1 + k_2)$$

where

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + h, y_i + hk_1)$$

Table of Difference formula			
First Derivative	2-Point	Forward	$f'(x) \approx \frac{f(x+h) - f(x)}{h}$
		Backward	$f'(x) \approx \frac{f(x) - f(x-h)}{h}$
	3-Point	Central	$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$
		Forward	$f'(x) \approx \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$
		Backward	$f'(x) \approx \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$
	5-Point		$f'(x) \approx \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$
Second Derivative	3-Point	Central	$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$
	5-Point		$f''(x) \approx \frac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2}$